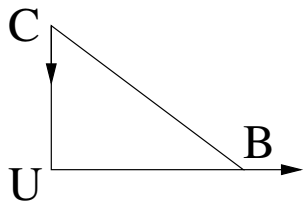
Answers for class prep quiz on section 3.9, Stewart's Calculus (8th ed.)

1. **Answer:** (d). In a related rates problem, we think of all quantities as functions of time t, and take derivatives with respect to t.

2. **Answer:** (a). By the product rule, the derivative of the left-hand side is
$$\frac{d}{dt}(PV) = P'V + PV',$$
and keeping in mind that n and R are constants, the derivative of the right-hand side is
$$\frac{d}{dt}(nRT) = nRT'.$$
(Note that (d) is a true statement, but is not simplified, since $R' = 0$.)

3. **Answer:** (c). Keeping in mind that m and g are constant, taking the derivative of both sides, we get
$$mg\frac{dh}{dt} + \frac{1}{2}m(2v)\frac{dv}{dt} = 0,$$
or
$$mg\frac{dh}{dt} + mva = 0,$$
where $a = v'$ is acceleration. Therefore,
$$\frac{dh}{dt} = -\frac{va}{g},$$
and velocity and acceleration are what we need to know to solve for $\dfrac{dh}{dt}$.



4. **Answer:** (d). Let $D(t)$ be the distance between the two trains, let $x(t)$ be the distance that Bart is east of Unpleasantville, and let $y(t)$ be the distance that Cal is north of Unpleasantville. By the Pythagorean theorem,

$$D(t) = \sqrt{x(t)^2 + y(t)^2},$$

so taking the derivative of both sides, we get

$$D'(t) = \frac{1}{2}(x(t)^2 + y(t)^2)^{-1/2}(2x'(t) + 2y'(t)) = \frac{x' + y'}{\sqrt{x^2 + y^2}}.$$

At the time in question, we have $x = 40$, $x' = +75$ (since x is increasing), $y = 30$, and $y' = -65$ (since y is decreasing), so

$$D'(t) = \frac{75 - 65}{\sqrt{40^2 + 30^2}} = \frac{10}{50} = +0.2.$$

Since $D'(t) > 0$, the distance between the trains is increasing, at a rate of 0.2 miles per hour.